



School District Educational Infrastructure and Change at Scale: Teacher Peer Interactions and Their Beliefs About Mathematics Instruction

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While current reform efforts press for ambitious changes to teachers' instructional practice, teachers' instructional beliefs are also consequential in such efforts as beliefs shape teachers' instructional practice and their responses to instructional reforms. This article examines the relationship between teachers' instructional ties and their beliefs about mathematics instruction in one school district working to transform its approach to elementary mathematics education. Quantitative results show that while teachers' beliefs did not predict with whom they interacted about mathematics instruction, teachers' interactions with peers about mathematics instruction were associated with changes in their beliefs over time. Qualitative analysis confirms and extends these findings, revealing how system-level changes in the district's

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educational infrastructure facilitated change in teachers' beliefs about mathematics instruction at scale.

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For a quarter century, school reformers have sought ambitious changes to classroom instruction, pressing to transform not only what content students should learn but also how they should learn it. These instructional reform initiatives are ambitious both in terms of the teaching and content they envision and by extension, the shifts in instructional practice they imagine. This is not new as school reformers have sought such ambitious transformations of modal instructional practice in U.S. classrooms for some time. During the 1920s and again in the 1960s, scholars proposed similar (if not equivalent) ideas for improving curriculum and instruction (Bruner, 1960; Dewey, 1938; Schwab & Brandwein, 1962).

Although current instructional reform efforts extend across subject areas, in this article, we focus on elementary school mathematics. Advanced by the National Council of Teachers of Mathematics (NCTM, 2000, 2014) as well as some state policymakers and the Common Core initiative of the National Governors Association Center for Best Practices and the Council of Chief State School Officers (2010), mathematics instructional reforms envision a classroom pedagogy that engages students in understanding the central concepts and processes of mathematics and applying such knowledge to real-world situations. A central component of the reformed instruction is making students' opportunities to learn mathematics more epistemologically authentic from a disciplinary perspective (Ball, 1993; Edelsky & Smith, 1984; Lampert, 1992).

With respect to content, reformers want *principled* mathematical knowledge to receive more attention. This is distinct from the *procedural* knowledge that historically has dominated the K–12 mathematics curriculum (Greeno, Riley, & Gelman, 1984; Lampert, 1986; Leinhardt & Smith, 1985; Romberg, 1984). While procedural knowledge centers on computation and following predetermined steps to compute answers to problems, principled knowledge focuses on the concepts that undergird mathematical procedures. Reformers argue that students should develop more sophisticated understandings of what it means to do mathematics, including solving mathematical problems, articulating conjectures, and reasoning with others about mathematical ideas, so that they come to appreciate mathematical activity as more than computation (Ball, 1993; Cobb et al., 1991; Fennema, Franke, Carpenter, & Carey, 1993; Lampert, 1992; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010;

NCTM, 1989, 1991, 2000, 2014; Thompson, 1992). In this article, we refer to these ideas as *reform-oriented* mathematics instructional approaches.

Given that procedural approaches continue to dominate U.S. teachers' mathematics instructional practices (Nesmith, 2008); thus, the implementation of reform-oriented practices necessitates sizable shifts in the mathematics content that is taught, the academic tasks on which students work, and the ways in which students interact with teachers and one another. For these ideals to be reflected in classrooms, most elementary teachers will have to change their practice a great deal, and school leaders and district policymakers will need to organize appropriate supports. Most teachers will need opportunities to engage in ongoing professional learning activities that develop their capacity to teach in ways that approximate the ambitious content and pedagogy advanced by reformers (Cohen & Barnes, 1993; Firestone, Fitz, & Broadfoot, 1999; Schifter & Fosnot, 1993; Spillane, 2006; S. M. Wilson, Peterson, Ball, & Cohen, 1996).

These opportunities necessarily involve facilitating changes in teachers' beliefs about mathematics content and the nature of mathematics instruction. Teachers' beliefs about instruction—both teaching and student learning—are an important consideration because they influence teachers' instructional practices (e.g., Pajares, 1992; Peterson, Fennema, Carpenter, & Loef, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984) as well as their responses to reform initiatives. Teachers' beliefs, for example, shape their implementation of instructional reforms (Spillane, Shirrell, & Hopkins, 2016) such that more traditional or procedural beliefs can undermine teachers' adoption of reform-oriented curricula (Coburn, 2001). Moreover, teachers' beliefs about mathematics have been associated with student learning in that subject (Polly et al., 2013).

Given the relevance of teachers' beliefs to their instructional practices and student achievement, we used a mixed-methods approach (Creswell & Clark, 2011) to investigate whether and how elementary teachers' beliefs about mathematics instruction changed over time as a reform-oriented curriculum was implemented in their school district. Because teachers' interactions can be important on-the-job opportunities to learn from peers, we explore the extent to which teachers' mathematics instructional interactions were associated with their beliefs about how elementary students should learn mathematics, and specifically if teachers' interactions were predicted by or predictive of their beliefs. Based on our findings, we argue that teachers' development of reform-oriented beliefs about mathematics instruction was facilitated by the district's educational infrastructure, which afforded teachers robust opportunities to interact with and learn from one another around the new curriculum.

Our account offers two contributions. First, we get inside the black box of teacher peer effects, showing that one way in which peers matter is in the extent to which they can shape each other's instructional beliefs. Opening

up this black box is critical to understanding the mechanisms that support change inside schools and how to develop appropriate professional learning opportunities for teachers. Second, whereas much of the prior literature on teachers' beliefs about mathematics instruction focuses on relatively small groups of teachers using focused interventions, we explore whether and how teachers' beliefs about mathematics instruction can be changed at scale or school district-wide.

Teachers' Beliefs and Change at Scale

We situate our study in research on teachers' beliefs about mathematics instruction. Then, we motivate our analysis using literature on school district educational infrastructure and teachers' instructional interactions. Finally, we present the communities of practice framework that informed our analysis.

Teachers' Mathematics Instructional Beliefs

Beliefs influence behavior (Abelson, 1979; Bandura, 1986; Nisbett & Ross, 1980). In general, beliefs are difficult to alter in adulthood, especially those that are integrated into a belief structure early in life (Lewis, 1990; Nespor, 1987). Future teachers' beliefs about instruction, for example, are well established by the time they enter college, often through an extended "apprenticeship of observation" to teaching, and tend to resist alteration (Buchmann, 1984, 1987; Clark, 1988; Floden, 1985; Lortie, 1975, p. 62; S. M. Wilson, 1990). Often, teachers "see what their beliefs allow them to see, that is, their beliefs act as a filter through which their observations are shaped" (Cooney & Wiegel, 2003, p. 800).

Teachers' instructional beliefs merit study because they influence teachers' planning, instructional decision making, and instructional practices (Pajares, 1992; Richardson, Anders, Tidwell, & Lloyd, 1991). Research has shown that teachers' beliefs about mathematics instruction influence their mathematics teaching practice (Beswick, 2012; Thompson, 1984), including their use of curricular materials (Clark & Peterson, 1986; Hart, 2002; Philipp, 2007; Romberg & Carpenter, 1986; Thompson, 1992; M. S. Wilson & Cooney, 2002) and their pedagogical approaches (Peterson et al., 1989; Stipek et al., 2001). For instance, whether teachers believe that mathematics is a set of procedures to be learned for generating correct answers or a tool for inquiry influences how they teach as well as how they assess mastery of students' learning (Stipek et al., 2001).

More generally, there is evidence to indicate that teachers' beliefs influence their responses to reform efforts. Beliefs about instruction in particular subject areas, for example, shape the ideas teachers construct about reform proposals and new curricula as well as whether and how they revise their instructional practice in response (*Educational Evaluation and Policy Analysis*, 1990; Jennings & Spillane, 1996; Peterson et al., 1989). While

most scholarship tends to depict changes in teachers' beliefs as preceding changes in practice, there is evidence to suggest that changes in beliefs can follow changes in practice, especially when "the complexities of classroom life . . . constrain teachers' abilities to attend to their beliefs" (Fang, 1996, p. 53; see also McLaughlin, 1990).

Changing teachers' mathematical instructional beliefs so that they align with reform-oriented views of mathematics instruction is difficult (Ball, 1990; Peterson et al., 1989; Schoenfeld, 1992; Williams & Baxter, 1996). Still, research on teachers' mathematical instructional beliefs reveals that these beliefs can and do change under certain conditions (Gill, Ashton, & Algina, 2004; Swars, Smith, Smith, & Hart, 2009). A series of intervention studies examining the effects of structured professional development using Cognitively Guided Instruction found that teachers who were engaged with the development of children's mathematical thinking transformed their instructional beliefs and that these changes persisted over time (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fennema et al., 1993, 1996; Franke, Carpenter, Levi, & Fennema, 2001; Peterson et al., 1989). In one three-year study, 17 of the 21 teachers involved in the professional development program came to believe, among other things, in children's ability to solve mathematical problems without being shown explicit procedures for solving them and articulated the value of having children use their own problem-solving strategies (Fennema et al., 1996). These findings are consistent with broader work on teacher professional development that suggests providing teachers with learning opportunities that focus on curriculum and content can enable change in their instructional beliefs, knowledge, and practices (Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hill, 2004; Hopkins, Spillane, Jakopovic, & Heaton, 2013; Parise & Spillane, 2010; Porter, Garet, Desimone, Yoon, & Birman, 2000).

Other studies also offer evidence that teachers' mathematical instructional beliefs are mutable. In an intervention study involving 161 prospective teachers, participants in a treatment group were sent activation messages intended to prompt recall of their prior mathematical instructional beliefs, then read "refutational texts" that challenged their traditional beliefs and outlined a rationale for adopting reform-oriented practices; in contrast, control group participants were provided a textbook on mathematics instruction (Gill et al., 2004). At the end of the study, teachers in the treatment group had developed more reform-oriented mathematical instructional beliefs than those in the control group. In another study involving 15 prospective teachers working in pairs with individual children on tasks designed to elicit children's mathematical thinking, results showed that participants' preexisting beliefs were challenged and they incrementally developed more reform-oriented beliefs (Ambrose, 2004).

Taken together, this evidence suggests that teachers, when provided resources and support, can develop more reform-oriented mathematical

instructional beliefs. Nonetheless, these findings derive from studies focused on small groups of teachers who voluntarily participated in professional learning interventions; thus, it is not clear whether or how teachers' mathematics instructional beliefs can change at scale or school district-wide.

Educational Infrastructure and Change at Scale

Examining change at scale requires attention to the role of the school district as the central organizing unit for the delivery of instruction in the United States (Gamson & Hodge, 2016; Honig, 2003; Spillane, 1998). As such, district leaders are primarily responsible for designing and developing educational infrastructures to implement and support instructional reform. By educational infrastructure, we mean those formal structures and resources intended to support teachers' instructional practice and enable efforts to improve that practice, including curricular materials, student assessments, and procedures and routines for analyzing practice and structuring the work of improvement (e.g., school improvement planning, coaching) (Cohen, Spillane, & Peurach, in press; Spillane, Hopkins, & Sweet, 2015).

A burgeoning body of literature examines how various components of a school district's educational infrastructure, either separately or together, influence instructional reform efforts on the ground. Findings from this work suggest that infrastructure components such as grade-level assignment, formal leadership positions, and participation in organizational routines like Professional Learning Communities (PLCs) can facilitate reform implementation when they foster increased and in-depth interactions among teachers about instruction and its improvement (Coburn, 2001; Coburn & Russell, 2008; Hopkins et al., 2013; Spillane et al., 2016; Stein & Coburn, 2008). For example, by introducing teacher leadership positions (e.g., coaches) or supporting structured time for grade-level team collaboration, districts and schools can enable teachers to share expertise, discuss new material and practices, and extend their learning from formal professional development (Brownell, Yeagar, Rennells, & Riley, 1997; Coburn & Russell, 2008; Penuel, Sun, Frank, & Gallagher, 2012; Spillane et al., 2016; Spillane, Kim, & Frank, 2012; Sun, Penuel, Frank, Gallagher, & Youngs, 2013; Sun, Wilhelm, Larson, & Frank, 2014). These types of interactions are important on-the-job learning opportunities that can influence teachers' knowledge development and their instructional practices and beliefs (Frank, Zhao, Penuel, Ellefson, & Porter, 2011; Penuel, Frank, Sun, Kim, & Singleton, 2013; Spillane et al., 2016; Sun et al., 2013), and they are positively associated with student achievement (R. Goddard, Goddard, Kim, & Miller, 2015; Ronfeldt, Farmer, McQueen, & Grissom, 2015). We thus anticipate that teachers' mathematical instructional beliefs will be associated with their access to other teachers' beliefs via their interactions:

Hypothesis 1: Teachers' mathematics instructional beliefs will be associated with the beliefs of those with whom they interact.

Although the extant literature suggests that the design and development of educational infrastructures can facilitate reform implementation by creating opportunities for teachers to learn from—and be influenced by—their peers, there is also evidence to suggest that teachers tend to interact with like-minded colleagues in making sense of instructional reforms (Coburn, 2001). Based on a study of first- and second-grade teachers in one California elementary school, Coburn (2001) documents how teachers tended to “self-select into informal networks with similar worldviews and approaches” about reading, which in turn contributed to the same messages about improving reading instruction being understood differently depending on teachers' networks (p. 156). Thus, while teachers may interact with peers more frequently as a result of redesigned educational infrastructures, the ideas they have access to via these interactions could potentially reaffirm their existing instructional beliefs, thereby limiting changes and their beliefs and thus their take-up of reforms. To explore whether the similarities among teachers in their beliefs about mathematics is due to social influence or social selection, we also consider whether teachers are more likely to interact with peers who have similar beliefs about mathematics instruction.

Hypothesis 2: Teachers are more likely to interact with peers who have similar beliefs about mathematics instruction.

Given the importance of educational infrastructure for supporting teachers' instructional interactions, we also sought to understand why we observed any associations between teachers' interactions and teachers' beliefs. Prior research reveals the limitations of teachers' interactions in enabling change when they contribute to “contrived collegiality” (Datnow, 2011; Hargreaves, 1994; Little, 1990) or “pseudo-communities” (Grossman, Wineburg, & Woolworth, 2001), which do little to facilitate teacher learning or change in their beliefs. We thus asked: How did the school district educational infrastructure for elementary mathematics education influence teachers' interactions about mathematics instruction? We describe the communities of practice framework that informed how we addressed this question next.

Teachers' Interactions in Communities of Practice

Though teachers' instructional interactions are important for providing on-the-job learning opportunities, the nature of these interactions is likely critical for whether or not they enable change in teachers' instructional beliefs (Coburn, 2001; Davis, 2003; Y. Goddard, Goddard, & Tschannen-Moran, 2007; Little, 2003; Siciliano, 2016; Smylie, 1995; Stein & Coburn, 2008). A

communities of practice (CP) framework is useful for considering the nature of interactions among teachers (Cobb, McClain, de Silva Lamberg, & Dean, 2003; Lave & Wenger, 1991; Stein & Coburn, 2008; Wenger, 1998).¹ A CP framework focuses on how participation in routine work practices can under certain conditions enable people to transform their practice (i.e., learn on the job) through participation with co-workers. Social interactions in CPs share three characteristics: *regular* engagement in a *common domain* using *shared practices* (Stein & Coburn, 2008; Wenger, McDermott, & Snyder, 2002). Whereas *common domain* refers to the “common ground” that brings participants together in a community of practice, *shared practices* refers to those “socially defined ways of doing things” or “common approaches” in which participants engage (Stein & Coburn, 2008, p. 595). CPs can develop through formal or designed organizational arrangements, informal networking, or some combination of both, and learning can occur within and between CPs.

As Stein and Coburn (2008) show, school districts can be made up of multiple and overlapping CPs, including district leadership, school leaders, coaches, and teachers. Critical to understanding learning within and between CPs in school districts are *boundary practices* that enable and sustain connections between different CPs and the boundary objects and boundary spanners that enable boundary crossing (Stein & Coburn, 2008; Wenger, 1998). *Boundary objects* denote reifications or artifacts (e.g., curriculum guidelines) that inhabit two or more communities of practice (Engestrom, 1987; Star & Griesemer, 1989; Wenger, 1998). *Boundary spanners* refer to individuals who participate in two or more CPs and thereby broker relations between these communities (Wenger, 1998). For example, coaches often participate in school teaching communities as well as district office curricular leadership communities (Coburn & Russell, 2008; Stein & Coburn, 2008). We draw on these constructs to theorize how the school district educational infrastructure shaped teachers’ interactions to enable or constrain shifts in their mathematical instructional beliefs.

Methods

We use data from a longitudinal project that was designed to understand how school staff interacted about literacy and mathematics instruction over time and explore relations between school staff interactions, organizational norms (e.g., trust), and teachers’ instructional beliefs. The project employed a sequential explanatory mixed-methods design (Creswell & Clark, 2011) in which we collected survey data each year from school staff in the district from 2010 to 2013, then conducted follow-up interviews with select teachers and leaders in 2011 and 2015.

Study Site

Our study includes all 14 elementary schools in one medium-sized school district that we call Auburn Park Public Schools. Auburn Park is a suburban district serving approximately 5,900 elementary students, the majority of whom are White (82%), with small populations of Latino/a (6%) and African American (5%) students. Approximately one-quarter of the district's elementary students receive free or reduced-price lunch.

After the statewide adoption of new K–12 mathematics standards in 2009,² Auburn Park leaders launched an initiative in fall 2010 to redesign the district's educational infrastructure in elementary (K–6) mathematics to support a reform-oriented instructional approach. These redesign efforts were led by a district-wide mathematics leadership committee, which was composed of teachers from schools across the district and included the adoption of *Investigations in Number, Data, and Space*, a curriculum supporting reform-oriented mathematics instructional practices (Slavin & Lake, 2008; TERC, 2013).

To support curriculum implementation efforts, district leaders also offered new or redesigned professional learning opportunities, including a master's program in mathematics education for select teachers that began in 2011. All 16 teachers who participated in the master's program were also members of the districtwide mathematics leadership committee, and 3 of these teachers became mathematics instructional coaches in 2012, who were each assigned to two schools. Within schools, district-mandated PLCs were redesigned to focus grade-level teams' efforts on instructional improvement through joint planning, problem solving, and analyses of student work (more on this in the following). Between schools, a new array structure brought teachers from schools with similar student populations and achievement levels together each quarter for targeted professional development.

Data Sources

Quantitative Data

All instructional staff filled out a School Staff Questionnaire each spring from 2010 to 2013. Response rates on the survey were as follows: 81% in 2010 ($n = 331$), 95% in 2011 ($n = 393$), 94% in 2012 ($n = 375$), and 94% in 2013 ($n = 384$). Our quantitative analysis focuses solely on the teachers in the sample and those who responded to the survey in at least three of four years ($n = 222$).³ To examine instructional interactions, the survey asked: "This school year, to whom have you turned to for advice and/or information about curriculum, teaching, and student learning?" as advice and information are considered fundamental building blocks for learning and knowledge development (Bransford, Brown, & Cocking, 2000). Respondents listed up to 12 individuals,⁴ and these names were auto-populated in a follow-up question that asked respondents to indicate the subject

for which they sought advice and/or information as well as the frequency with which they sought advice and/or information from that person in the last year: a few times per year = 1, monthly = 2, weekly = 3, and daily = 4. The present analysis focuses solely on mathematics-related interactions.

Teachers' beliefs about mathematics instruction were assessed using the Mathematics Attitude Inventory, an 18-item scale adapted from the Fennema-Sherman Short Form for use with teachers (Capraro, 2001; Mulhern & Rae, 1998). Teachers indicated agreement to each item on a 5-point Likert scale, from *strongly disagree* to *strongly agree* (see Appendix). Using exploratory factor analyses, we found that 11 of the 18 items loaded onto two factors that measured distinct aspects of teachers' instructional beliefs. The first factor comprises six items focused on how students should learn mathematics or whether students should first acquire procedural or principled mathematical knowledge. The second factor comprises five items related to how teachers should facilitate mathematics instruction. Because our analysis revealed change over time in the first factor but not the second and we were interested in understanding the longitudinal relationship between teachers' beliefs and interactions, our analysis focused on the first factor. To facilitate more intuitive interpretation, we reversed the scale for each item so that a higher score indicated more reform-oriented beliefs.

We also used several binary or categorical survey items, including gender and race/ethnicity, years of teaching experience, grade-level assignment, leadership roles, and hours of professional development in mathematics.

Qualitative Data

We conducted semi-structured interviews at two timepoints with a purposeful sample (Lincoln & Guba, 1985; Patton, 1990) of leaders and teachers in five elementary schools. We selected schools for inclusion in the sample along dimensions believed important for facilitating instructional interactions, such as the presence or absence of a mathematics coach or teacher leaders (Hopkins et al., 2013; Sun et al., 2013). Of our qualitative sample of five elementary schools, four schools had a part-time mathematics coach, two schools had mathematics teacher leaders who participated in the master's program, and one school had neither a coach nor a teacher leader. Selecting schools in this way allowed for the identification of patterns that held across schools that differed along these dimensions. We also interviewed the district's elementary curriculum director and two instructional facilitators who led the districtwide mathematics leadership committee to better understand the educational infrastructure from the district perspective.

Within schools, we selected teachers and leaders who held different positions and were both highly and weakly connected in their school's mathematics advice and information network. We interviewed the principal, the mathematics coach and/or teacher leaders (when applicable), and

Table 1
Interview Sample by Position Over Time

	2011–2012	2014–2015	Both Years
District leaders	2	3	2
Principals	5	5	4
Literacy coaches	3	2	2
Math coaches	3	3	3
Math teacher leaders ^a	2	2	2
Teachers	18	16	10
Total	33	31	23

^aMath teacher leaders are classroom teachers who are members of the district mathematics leadership committee; they do not hold coach positions.

between three and five other staff members representing different positions (e.g., special and general education, literacy coaches), grade levels, and network connectedness. During our first round of interviews in 2011–2012, our sample included 33 individuals: 2 district office staff, 5 principals, 3 literacy coaches, 3 mathematics coaches, 2 mathematics teacher leaders (non-coaches who served on the district leadership committee), and 18 teachers (see Table 1). In 2014–2015, 23 of the interviewees from the first round were interviewed again, including 3 district office staff, 4 principals, 3 literacy coaches, all math coaches and teacher leaders, and 10 teachers. If an interviewee had left the district, we replaced them with someone in a similar position in their school and network; this process led to the inclusion of 1 new principal and 5 new teachers. We also interviewed an additional district office staff member, for a total sample of 31 interviewees at the second timepoint.

In both rounds of interview, we asked participants with whom they interacted about mathematics instruction, how and why they interacted, and to describe the nature and content of those interactions. Given that our interest in teachers' beliefs emerged from ongoing analysis, in 2014–2015, we asked interviewees to reflect on whether and how their peers had influenced their ideas about and understandings of the new curriculum. Interviews lasted between 40 and 50 minutes and were audio recorded, transcribed, and imported to NVivo for analysis.

Analytical Procedures

Quantitative Analysis

We explored our first hypothesis related to whether teachers' beliefs were associated with the beliefs of peers with whom they interact by fitting multilevel growth models for change using a person-period data set (i.e.,

Level 1 was within-teacher change over time; Level 2 was between-teacher change across years; Singer & Willett, 2003). The dependent variable was teachers' beliefs about mathematics instruction; independent variables included year, access to peers' beliefs about mathematics instruction, years of experience, highest grade taught, hours of math professional development, and teacher leadership role.

We measured the extent to which a teacher had opportunities to engage with their peers' beliefs about math instruction via their interactions using an approach similar to Frank, Zhao, and Borman (2004); Penuel, Sun, Frank, and Gallagher (2012); and Sun et al. (2013). While these scholars use the term *exposure* to refer to this phenomenon, we use the term *access* to underscore the extent to which teachers' interactions facilitated their access to peers' beliefs about mathematics instruction. First, we multiplied the frequency of the interaction teacher i reported with teacher j at Time 2 by teacher j 's average beliefs at Time 1.⁵ Then, we calculated the sum of each teacher's access to his or her peers' beliefs at Time 2:

$$\text{Access to peers beliefs}_i = \sum_{\substack{i'=1, \\ i \neq i'}}^{n_i} (\text{frequency of interaction}_j) \times (\text{mathematics instructional beliefs}_j).$$

To control for individual differences in network size, we divided each sum by the size of teacher i 's network at Time 2. Given the time lag in access to peers' beliefs, we calculated this variable only for 2011, 2012, and 2013.

An examination of empirical growth plots for beliefs across the years of our study provided support for a linear relationship. We thus fit an unconditional growth model, where time was the only fixed effect, to determine if beliefs changed significantly over time. Since access to peers' beliefs was a time-varying predictor, we then fit a conditional model that included both time and access as fixed effects to determine whether access to peers' beliefs at Time 1 was associated with teachers' beliefs at Time 2. For ease of interpretation, we standardized the access and belief variables. Next, we added main effects for all time-invariant predictors to determine whether they were associated with average beliefs at Time 1 as well as interaction terms (Predictor \times Time) to assess whether they were associated with change in beliefs over time. Of these predictors, only the main effect of years of experience was significant and thus is the only independent variable other than access to peers' beliefs presented in the following.

The final composite multilevel model for change is given as:

$$Y_{ij} = [\gamma_{00} + \gamma_{10} \text{TIME}_{ij} + \gamma_{20} \text{EXPOSURE}_{ij} + \gamma_{01} \text{YEARSEXP}_i] + [\zeta_{0i} + \zeta_{1i} \text{TIME}_{ij} + \varepsilon_{ij}].$$

In this equation, Y_{ij} is teacher i 's belief about mathematics in year j , γ_{00} represents the population initial average in spring 2011, and γ_{10} represents the

average yearly rate of change in beliefs, controlling for access to peers' beliefs and years of teaching experience. Next, γ_{20} is the average difference at any given time in beliefs between teachers who were one standard deviation apart in their access to peers' beliefs, and γ_{01} represents the effect of years of experience on teachers' initial beliefs in spring 2011. Finally, ζ_{0i} and ζ_{1i} are the Level 2 residuals that represent any unexplained portions of initial status or rate of change, and ε_{ij} is the portion of individual i 's outcome that is unpredicted by occasion j .⁶

To examine our second hypothesis related to whether teachers were more likely to have ties with teachers whose beliefs were similar to their own, we use hierarchical latent space models (HLSMs; Sweet, Thomas, & Junker, 2013). Because social interactions, or ties, are not independent, specific models are needed to relate individual attributes with ties and HLSMs accommodate network structure through the use of latent positions. Latent space models assume that individuals, or nodes, in a network each have a position in a latent social space and the probability of a tie between any pair of nodes is a function of the distance between them in the latent space. Thus, the dependent variable for our HLSMs was the presence of a tie between two teachers. For every pair of teachers i and j , if i turned to j for advice, the $i \rightarrow j$ relationship was assigned a value of 1. If i did not turn to j for advice, the relationship was assigned a value of 0. The model is given as:

$$\begin{aligned} \text{logit } P(Y_{ijk}=1) & \\ & = \beta_0 + \beta_1 \text{SLEAD} + \beta_2 \text{SMULTI} + \beta_3 \text{RLEAD} + \beta_4 \text{MULTI} + \beta_5 \text{GRADE} \\ & + \beta_6 \text{BELIEFS} - |Z_{ik} - Z_{jk}|, \end{aligned}$$

where $Y_{ijk}=1$ indicates a tie from teacher i to teacher j in school k and Z_{ik} is the latent position of teacher i in school k . We included six independent variables: four individual-level covariates to examine whether being a teacher leader (i.e., member of the district leadership committee) or teaching multiple grades was related to sending and receiving advice or information (SLEAD, RLEAD, SMULTI, and RMULTI) and two dyad-level measures to explore the extent to which teaching the same grade and having similar beliefs about mathematics⁷ were related to the presence of a tie between two teachers (GRADE and BELIEFS). Because HLSMs relate covariates to network ties for cross-sectional data, we fit separate HLSMs for each year.⁸

All quantitative models included the 222 teachers from whom we had survey responses for at least three timepoints between 2010 and 2013. About 45% of these teachers taught Grades K–2, and the remaining taught Grades 3 through 6. Approximately 20% had fewer than 5 years of experience, and about 40% had between 6 and 15 years of experience.

Table 2
Teacher Interview Sample Detail, 2011–2012 and 2014–2015

Name	School ^a	Position
Less reform-oriented		
Joanne	Bryant	Grade 6
Rebecca	Chavez	Grade 5
Evelyn	Kingsley	Gifted (Grades 2–6)
More reform-oriented		
Tracy	Ashton	Special education (Grades 2, 4, 5)
Rachel	Chamberlain	Kindergarten
Jessica	Chamberlain	Grade 2
Loretta	Chamberlain	Grade 3
Becky ^b	Chamberlain	Grade 4
Katie	Chavez	Grade 6
Jodie ^b	Chavez	Special education (Grades 2, 4)
Laura	Kingsley	Grade 3
Clarissa	Kingsley	Grade 1

^aAshton, Bryant, Chamberlain, and Chavez had part-time math coaches; Ashton and Chamberlain also had math teacher leaders. Kingsley did not have a math coach or a math teacher leader.

^bMath teacher leader.

Qualitative Analysis

We analyzed our interview data in four phases using open and closed coding to address our research question related to how the educational infrastructure shaped teachers' interactions. In Phase 1, two researchers independently coded two interviews for the how, why, and what of teachers' interactions about mathematics instruction, then met to discuss emergent themes within these categories and develop an initial closed coding scheme. To establish interrater reliability, in Phase 2, the researchers independently coded one-third of the interviews, met to discuss any coding disagreements and add codes as necessary, and recoded the data until kappa coefficients ranging from .72 to .99 were achieved (Carey, Morgan, & Oxtoby, 1996). In Phase 3, one researcher coded the remainder of the data and wrote analytic memos about how components of the educational infrastructure, along with other factors, shaped teachers' interactions as well as the substance of those interactions. Additionally, we used various functions in NVivo to identify the prominence of particular codes by school and position (e.g., principal, coach, teacher) and generate reports for analysis at the intersection of different how, why, and what codes. In the final phase of qualitative analysis, we drew on our quantitative findings to identify teachers for whom we had interview data at both timepoints and whose beliefs became more reform-oriented ($n = 9$) as well as those whose beliefs became less

Table 3
Teachers’ Mean (SD) Beliefs About Mathematics Instruction, 2010–2013

	All Teachers (<i>n</i> = 222)	Teachers on Mathematics Leadership Committee (<i>n</i> = 16)	Other Teachers (<i>n</i> = 206)
2010	3.212 (0.731)	4.111 (0.664)	3.212 (0.718)
2011	3.352 (0.829)	4.217 (0.321)	3.333 (0.817)
2012	3.484 (0.821)	4.246 (0.355)	3.356 (0.785)
2013	3.537 (0.762)	4.306 (0.414)	3.409 (0.755)

reform-oriented (*n* = 3) over time. This group included 10 teachers and 2 teacher leaders who were either general or special education teachers in Grades K–6 and who represented all five schools (see Table 2). We generated separate reports for the how, what, and why codes of each group, comparing and contrasting their interactions to theorize about what might account for differences in the direction of change in their beliefs.

Findings

We first explore our two hypotheses related to relations between teachers’ mathematics instructional beliefs and their interactions. Then, to explain and extend our quantitative results, we use our qualitative data to consider how various components of the educational infrastructure worked together to focus teachers’ interactions about improving mathematics instruction.

Teachers’ Beliefs and Interactions Related to Mathematics Instruction

Over the time of our study, teachers’ beliefs about how elementary students should learn mathematics changed to align with more reform-oriented approaches, from an average of 3.21 out of 5 in 2010 to 3.54 in 2013, a modest but statistically significant change (see Table 3). While teachers on the district mathematics leadership committee reported more reform-oriented beliefs than other teachers across all years, the magnitude of change in their average beliefs was the same for teachers who did not serve on the committee. In general, a majority of teachers in the overall sample (60%) developed more reform-oriented beliefs over time compared to about 30% of teachers who shifted toward less reform-oriented beliefs and 10% whose beliefs did not change.

To understand whether and how teachers’ beliefs were associated with their interactions across the years of our study, we explored findings related to our two hypotheses. First, we examined whether or not teachers’ mathematics instructional beliefs were associated with their access to their peers’ beliefs from 2010 to 2013. Similar to our descriptive findings in Table 3, the unconditional growth model indicated that changes in teachers’

Table 4
Results From Multilevel Models for Change in Teachers' Beliefs (n = 222)

		Model A	Model B	Model C
Fixed effects				
Composite model	Intercept	-0.142	-0.053	-0.296
	(initial status)	(0.076)	(0.107)	(0.138)
	Year	0.051*	0.024	0.023
	(rate of change)	(0.022)	(0.031)	(0.030)
	Access to peer beliefs		0.076*	0.081*
	Years of experience			-0.017** (0.006)
Variance components				
Level 1	Within-person	0.371 (0.024)	0.377 (0.028)	0.377 (0.028)
Level 2	In initial status	0.554 (0.074)	0.622 (0.100)	0.588 (0.097)
	In rate of change	0.011 (0.006)	0.001 (0.008)	0.001 (0.008)
Goodness-of-fit statistics				
	Akaike Information Criterion	1,952.57	1,580.32	1,574.89
	Bayesian Information Criterion	1,976.02	1,607.03	1,606.05

Note. Standard deviations in parentheses.
 * $p < .05$. ** $p < .01$.

mathematics instruction beliefs over time were significant, where a one-year change was positively associated with a 0.05 standard deviation change in teachers' beliefs (see Model A in Table 4). After including access to peers' beliefs about mathematics instruction as a time-varying predictor, we found a significant and positive association between teachers' beliefs and their access to peers' beliefs (see Model B). This finding suggests that controlling for the linear time trend, teachers indicated more reform-oriented beliefs about mathematics instruction when they interacted with peers who reported more reform-oriented beliefs and less reform-oriented beliefs when they interacted with peers who reported less reform-oriented beliefs. To quantify this relationship, a 1 standard deviation unit increase in access to peers' reform-oriented beliefs in a particular year was associated with a 0.08 standard deviation change toward more reform-oriented beliefs in the next year (see Models B and C). Although this is a small statistically significant

Table 5

Hierarchical Latent Space Models Coefficient Effect Estimates for 2011, 2012, 2013 Advice and Information Ties Related to Mathematics Instruction

Covariate		2011	2012	2013
Seeker	Leader	0.20	0.31	0.20
	Multiple grades	-1.09*	-1.82*	-1.44*
Provider	Leader	2.00*	2.00*	1.68*
	Multiple grades	-1.01*	-1.66*	-1.44*
Dyad	Same grade	4.04*	3.67*	4.56*
	Dissimilar beliefs	-0.01	0.03	-0.01

Note. Significance determined by 95% credible interval not containing 0.
* $p < .05$.

effect, it is notable given that we observed this effect at scale, or across all Auburn Park teachers.

The only other predictor that was significantly associated with teachers' beliefs about mathematics instruction in our longitudinal growth models was teaching experience (see Model C⁹). This relationship indicated that at initial status, teachers with more years of experience had slightly less reform-oriented beliefs on average, where every additional year of teaching experience was associated with a .02 standard deviation change toward less reform-oriented beliefs. Hours of mathematics professional development, leadership position, and grade level, however, were not significant predictors of either initial status or rate of change in teachers' beliefs (not shown in Table 4 for brevity); nonetheless, the effect of having access to peers' beliefs remained significant after controlling for these variables.

Second, we found that teachers in Auburn Park were not more likely to interact with peers who held similar beliefs about mathematics instruction than other Auburn Park teachers (see Table 5).¹⁰ However, teaching the same grade level had a positive and significant effect on teachers' interactions, as indicated by a large, positive dyadic effect of same grade: 4.04 in 2011, 3.67 in 2012, and 4.56 in 2013.¹¹ Additionally, teachers were more likely to turn to individuals who held formal leadership positions for advice and information about mathematics instruction, such as teachers who held positions on the district mathematics leadership committee. These findings suggest that while Auburn Park teachers did not tend to interact with like-minded colleagues about mathematics instruction, components of the district's educational infrastructure (e.g., grade level, formal positions) were important for shaping teachers' mathematics instructional interactions.

Our quantitative findings revealed that while Auburn Park teachers did not tend to interact with peers who held similar mathematics instructional beliefs during the time of our study, teachers' beliefs were associated with

the beliefs of peers with whom they interacted and teachers' beliefs became more reform-oriented over time on average. These results are somewhat surprising for two reasons. First, contrary to prior scholarship (Coburn, 2001), we found no evidence that teachers tended to interact with like-minded colleagues to share information about mathematics instruction. Second, some research suggests that teachers' interactions with peers can be powerful resources for improvement (Daly, 2010; Penuel et al., 2013), whereas other studies point to the limitations of such interactions for enabling reform (Hargreaves, 1994). We turn to our qualitative analysis to understand what might account for teachers' interactions being associated with changes in their mathematics instructional beliefs.

Teachers' Beliefs, Interactions, and Educational Infrastructure

We develop and support two assertions based on our analysis of qualitative data. First, we theorize how core components of the district's educational infrastructure facilitated interactions within and between different communities of practice that focused teachers' attention on implementing the new curriculum. We argue that these interactions were supported by boundary objects, boundary practices, and boundary spanners and required Auburn Park teachers to engage in ways that, *on average*, influenced their development of reform-oriented beliefs. Second, we compare and contrast teachers' interactions between those whose beliefs became more reform-oriented and those whose beliefs became less reform-oriented over time and theorize that the substance and depth of the interactions were influential in shaping the direction of change in their beliefs over time. (See Table 2 for a list of interviewees and their beliefs-related characteristics.)

Educational Infrastructure and Teachers' Interactions in Communities of Practice

Following Stein and Coburn's (2008) operationalization, we delineated four overlapping CPs where Auburn Park staff engaged with colleagues at least monthly (regular interactions) about mathematics instruction and its improvement (shared domain) using common practices such as planning units, analyzing students' mathematics work, and identifying and problem-solving challenges of teaching the new curriculum (shared practices): (a) district mathematics leaders, (b) school principals, (c) math coaches and teacher leaders, and (d) teachers who taught mathematics, including general and special education teachers in Grades K–6 (see Figure 1).

As depicted in Figure 1, several components of Auburn Park's infrastructure for elementary mathematics education worked as boundary practices that connected different CPs, including the mathematics leadership committee, school improvement teams, within- and between-school professional development, and PLCs. Our analysis focused chiefly on two of these boundary

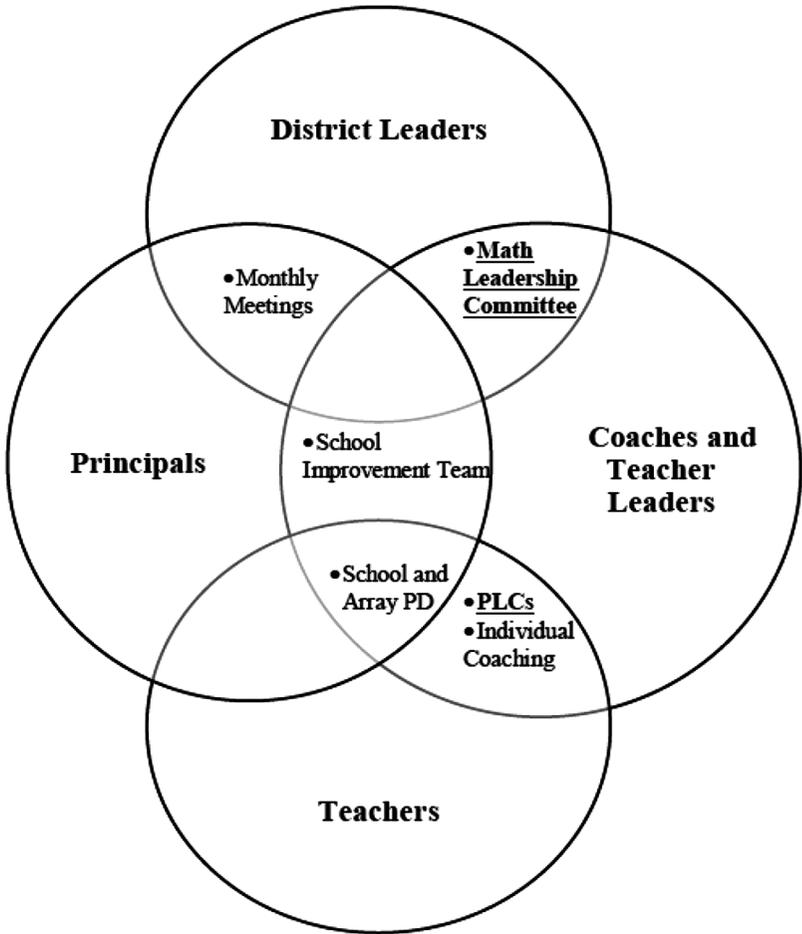


Figure 1. Boundary practices related to elementary mathematics instruction at Auburn Park.

practices that featured prominently in all interviewees' talk about mathematics instruction and its improvement: the mathematics leadership committee and PLCs. Whereas the former connected district leadership with mathematics coaches and other teacher leaders, the latter connected coaches and teacher leaders with teachers of mathematics. Focusing on these two boundary practices and their associated boundary objects and boundary spanners, we explore why teachers' beliefs became more reform-oriented on average

over time in Auburn Park as well as why teachers' access to peers' beliefs was associated with their own mathematics instructional beliefs.

District mathematics leadership committee as a boundary practice. District leaders intentionally selected teachers to serve on the mathematics leadership committee who were known as informal math leaders in their schools. Committee members reported at least monthly participation in formal meetings in which they interacted with district leaders to develop standards-based mathematics instructional units and assessments aligned with *Investigations*, modify these units and assessments as necessary as challenges were identified by teachers at their school sites, and design mathematics professional development for schools and arrays to support curriculum implementation. District leaders supported teachers' capacity to take up this work by providing opportunities to learn about reform-oriented approaches to teaching mathematics. Mary, a teacher who served on the committee and later became a math coach, noted how influential these supports were, which were based on the principles and practices supported by the NCTM: "We studied NCTM's principles, and that was when [the committee's] philosophy began to change to focus on their five practices—reasoning, representing, communication [*sic*]." As the committee's philosophy changed to a more reform-based approach, district leaders offered members the opportunity to participate in the master's program at the local university, which Mary noted both supported and extended the committee's work:

Then the master's program came at a perfect time because the district wanted to shift in that direction. We began that process together, knowing that math was much more than just showing kids what to do. We began to make that shift. Then we went through the program and continued to deepen our understanding and philosophical change.

By selecting leadership committee members for participation in the master's program, Auburn Park leaders ensured that those involved with curricular development would have opportunities to acquire the knowledge and skills—and to develop the beliefs—needed to support a reform-oriented approach to teaching mathematics. As we discuss in the next section, the materials and resources developed by the committee became boundary objects that when taken up in school PLCs influenced teachers' interactions about instruction.

District leaders intended committee members to serve as boundary spanners connecting the curriculum they selected to schools: "The curriculum will be pushed out as a district, and when I say district it will come from the leadership committee, [including] math coaches, and then pushed out into the buildings" (Georgia, elementary curriculum director). Committee members understood their boundary spanning role as one member described it, "to come back and . . . be the math leaders in our building" (Emily, committee member

and math coach). Part of their work as math leaders included developing teachers' capacity to implement the new curriculum via professional development as well as participation in PLCs. As Jodie, a special education teacher and committee member, shared:

We're given a lot of training in the committee that we're expected to bring back to the buildings, and so we hear about a lot of things . . . in committee meetings . . . I think that deepens the understanding and kind of the light bulb goes on of, "These are things I need to be doing."

The committee thus served as a boundary practice that supported teachers in developing knowledge specific to implementing reform-oriented mathematics instruction, helping them "deepen their understandings" and identify the "things they need to be doing" to support other teachers at their schools. Indeed, Emily explained how committee members worked together to figure out "how to facilitate . . . how to help teachers understand why we're not doing algorithms so much and we're doing the student-invented strategies more."

The committee also facilitated bidirectional coordination between teachers and district leaders. An explicit aspect of committee members' work in schools was to gather feedback from teachers about curriculum implementation and bring it back to the committee to address. Katie, a Grade 6 teacher, explained how her grade-level peer who served on the committee was her "lifeline" to get information about mathematics instruction to the district office:

I talk to . . . the other sixth grade teacher because she is on the committee . . . that's who I go to because she's kind of the lifeline to the curriculum department at central office. I'll tell her that I really struggled with [the curriculum unit].

Overall, the mathematics leadership committee provided members with opportunities to learn about reform-oriented approaches, apply that learning in the development of curriculum resources and materials, and share these boundary objects with teachers in schools. As such, the committee was a boundary practice crossing boundaries of communities of practice to support the implementation of reform-oriented mathematics instruction. Committee members' interactions with teachers were also enabled by PLCs, another boundary practice that enabled mutual engagement with boundary objects.

Professional learning communities as a boundary practice. In both 2011 and 2015, all interviewees consistently referenced regular participation in PLCs, either weekly or bimonthly. PLCs were comprised of grade-level teachers—both general and special education—as well as mathematics leadership committee members who taught at that grade level and math coaches who were members of all PLCs in their two schools. Principals attended PLCs

sometimes to observe or in an advisory role but did not engage as regularly as others.

Teachers' descriptions of PLC practices suggested that they involved deliberations with peers about implementing the new curriculum and more broadly efforts to adopt a reform-oriented approach to teaching mathematics. As teachers worked to understand and use curricular boundary objects—the *Investigations* materials and associated units and assessments developed by the district leadership committee—their long-standing ideas about mathematics instruction were disrupted because these objects represented a substantively different approach than prior curricula in terms of what it meant for students to learn mathematics: “It’s just so different [*Investigations*], the investigating part of it,” Andrea, a Grade 3 teacher explained. “You know, just trying to make sure, ‘Golly, tell me again, what’s the purpose of this?’ . . . Some of them don’t really kind of make sense to me.” This difference created some dissonance, another interviewee explained, as teachers were:

struggling with it . . . they weren’t taught that way. They were taught by using algorithms, so this is a whole new way of thinking, so they’re feeling a little bit of discomfort. In the past it was like, let’s open the book, go through the lesson. . . . This is a whole different way of teaching. (Sue, literacy coach)

Teachers’ accounts suggest that the discomfort they felt motivated them to engage and interact in PLCs as they sought opportunities to grapple with the instructional decisions and challenges they faced:

When we had just the regular curriculum, there really wasn’t much to discuss. We were both on page 20. Whereas now there’s so many different strategies and things people are doing to help emphasize *Investigations*. I definitely think there’s more room for discussion than there used to be. (Lucy, a kindergarten teacher)

Similar to Lucy, all interviewees observed, unprompted, that the *Investigations* materials and their accompanying units and assessments pressed teachers to interact with peers to make sense of the new approach to teaching mathematics. Thus, the curriculum materials were boundary objects that traveled from the district leadership committee into schools, figuring prominently in PLCs and shifting teachers’ discussions about teaching mathematics.

An example of a shared practice that took place in PLCs as teachers implemented the new curriculum was the “double scoring” of students’ mathematics writing on district assessments. This practice not only required teachers to develop shared understandings of the math assessments but also how students should demonstrate their mathematical learning. As Carmen, a Grade 5 teacher, explained, her PLC peers helped her think more deeply

about what counted as “appropriate” student work in the context of the new curriculum:

They’re helping me think through, “Is this an appropriate response for a fifth grader?” Sometimes I think it’s not, but she’ll point out, “But they did this and this” and I’ll have not thought about that, like, “They used this vocabulary well” . . . helping me analyze student responses and just show understanding.

Indeed, the majority of teachers we spoke with described developing new understandings of students’ mathematical thinking from discussing the scoring of student assessment during PLCs. Focusing on students’ mathematical work during PLC deliberations is noteworthy as research shows that teachers often change their beliefs about mathematics instruction when they attend to the development of students’ mathematical thinking (Franke et al., 2001).

Teachers’ interactions in PLCs were also enabled by boundary spanners, with three-quarters of the teachers we interviewed identifying members of the district mathematics leadership committee, and especially mathematics coaches, as individuals who enabled their exchanges about mathematics instruction with peers. Rachel, a kindergarten teacher, explained how the math coach at her school helped her PLC understand the rationale and objectives for various lessons used to supported *Investigations*:

Our [grade] team plans and we get to collaborate together. Our math coach . . . when we’re planning together, if we have a question she’s always there to help . . . she knows a lot . . . more about the curriculum . . . the reason behind . . . the math. . . . [She tells us,] “This is what you really wanna have the kids get out of this.”

In addition to serving as resources during PLCs to help explain aspects of the new curriculum, coaches facilitated interactions among teachers. For example, Abby described how she supported a Grade 1 teacher when one of her grade-level colleagues began to question her use of a reform-oriented instructional strategy for developing number sense:

One element of her routine is domino builder, and I let her take it on [in her PLC]. One of her team members said, “Well, do you really find any benefit to it?” Then she started talking, and I went in and supported her. But, she was able to support and explain why it’s so important, and how it’s that instant knowledge of what those numbers are instead of having to count, and how we want to move kids into that. Then we talked about how that is so strong in building what they need in first grade.

Here, Abby describes how she brokered the development of shared understandings between PLC members around the importance of using reform-oriented instructional approaches.

The Limits of the Educational Infrastructure

Our account thus far documents how two boundary practices—the district leadership committee and PLCs—brought members of different communities of practice (district leaders, teacher leaders, teachers) together in ways that facilitated teachers’ interactions about the implementation of a reform-oriented approach to teaching mathematics. Indeed, the shared understandings developed via engagement in these boundary practices help to explain why we observed an overall positive shift in teachers’ beliefs about mathematics over time in Auburn Park. Nonetheless, our quantitative analyses also showed that nearly one-third of teachers in our sample developed less reform-oriented beliefs over the course of our study. To understand why the aforementioned boundary practices may have enabled some teachers but not others to develop more reform-oriented mathematics instructional beliefs, we compared and contrasted interview data from the 10 teachers whose beliefs became more reform-oriented over time to the 3 teachers whose beliefs became less reform-oriented (see Table 2).

Teachers in both groups did not report any differences in terms of the regularity of their participation in PLCs. Further, they reported engaging in the same common practices such as discussing teaching strategies, planning lessons, and double scoring students’ mathematical work. In addition, teachers in both groups reported interacting regularly with boundary spanners (i.e., math coaches or teacher leaders) about mathematics instruction. Even teachers at Kingsley, where no math coach or teacher leader was present, described accessing coaches and teacher leaders via the array structure that brought schools together for professional development.

Given no readily apparent differences in teachers’ frequency of interactions in boundary practices or with boundary spanners, we explored whether or not some teachers were simply more resistant to change than others. We found that all three teachers whose beliefs became less reform-oriented expressed some reservations about the *Investigations* curriculum and in particular the lack of attention to mathematical facts. Rebecca, a Grade 5 teacher, for example, noted: “I like *Investigations*, I like the concepts, I like the discussion, I like the strategies . . . but it’s not complete.” Joanne, a Grade 6 teacher, likewise noted being “skeptical about why am I doing” mathematics in the manner emphasized by *Investigations*: “Sometimes I just have to trust *Investigations* to think that the authors knew what they were doing. I might not always agree or see the point.” Nonetheless, five of the nine teachers who developed more reform-oriented beliefs expressed similar reservations. For example, Jessica, who described herself as “set in her ways” and as having “strong beliefs” about the importance of memorizing mathematical facts, explained that “the biggest holes” in *Investigations* are lack of attention to “math facts.” She went on to note:

I look at the studies that say that if you don't know your math facts, how can you do Algebra later? . . . I really think it's important for them to learn their math facts. I feel like my kids don't know them well enough, and in second grade you shouldn't be counting on your fingers.

Convinced that *Investigations* did not focus enough on mathematical facts, she described supplementing the curriculum by explicitly teaching math facts. Although Jessica claimed, "It's hard to change my belief" about math, she did develop more reform-oriented beliefs over time. Indeed, seven of the nine teachers who developed more reform-oriented beliefs spoke explicitly about the initial challenges of learning to teach mathematics using the new curriculum.

One difference between teachers whose beliefs became less and more reform-oriented over time, however, was that two of the three teachers (Joanne and Rebecca) who developed less reform-oriented beliefs interacted each year with peers who, on average, reported less reform-oriented beliefs about mathematics instruction. The third teacher (Evelyn) interacted with peers with both above and below average reform-oriented beliefs. In contrast, the nine teachers who developed more reform-oriented beliefs overwhelmingly interacted with peers who reported more reform-oriented beliefs about mathematics instruction across all years, with the exception of two teachers in one study year.

Another difference between teachers who developed more and less reform-oriented beliefs over time was related to the substance of their exchanges about mathematics instruction with peers, including boundary spanners. Whereas teachers in both groups reported engaging in conversations that focused on surface-level aspects of mathematics instruction (e.g., discrete classroom activities and teaching strategies) and logistics (e.g., finding materials, pacing), only those who developed more reform-oriented beliefs reported participating in interactions that focused on deeper pedagogical and/or mathematical principles that motivated and undergird these activities and strategies. Prior research documents differences in the substance of teachers' interactions about instruction and associates these differences with teachers' success in adopting more reform-oriented mathematics instruction (Coburn, 2004; Spillane, 1999, 2000; Stein & Coburn, 2008; Stein, Grover, & Henningsen, 1996).

All three teachers who developed less reform-oriented beliefs described discussions with peers about *Investigations* that focused exclusively on either logistics and/or surface aspects of instruction. Evelyn (gifted education teacher), for example, noted: "We always talk about math, we always talk about just strategies and kids and things like that." Similarly, Rebecca described her interactions with a grade-level peer who is "more game oriented," noting that their exchanges prompted her to consider using more

games in her mathematics lessons: “I should maybe turn that into a game instead . . . I try to remember to do things a little bit differently than my natural tendency.” Four years later, Rebecca’s accounts of her exchanges with colleagues continued to focus on surface-level features of mathematics instruction, where she described how her colleague “likes competition and games, and I don’t like competition and games. But the kids like competition and games and so I can be more like that.” Joanne also described exchanges with her peers that exclusively focused on surface features of instruction, including “templates,” “checklists,” and “puzzles” to use with her students. In 2011, she explained: “We talk about how to do the investigation, how are you going to be doing this and pacing and how much time are you spending on this part of it to fit everything in.” Four years later, Joanne reported continuing to work with grade-level peers around mathematics instruction, noting how a colleague,

just gave me another math thing [activity] yesterday; “I worked on this coin counter thing.” . . . He was saying “I was trying to figure out how to do this with my class so I made a little template” and so he gave it to me. And it was where the kids fill in so that we can have some puzzles for math.

Further, in 2015, both Joanne and Rebecca described their PLCs’ interactions with math coaches as focused on either logistics or strategies, with Joanne saying, “She helps us, she does the logistics,” and Rebecca stating, “She might say ‘Oh, I saw somebody do this,’ so she has strategies to share.”

Whereas all nine teachers who developed more reform-oriented beliefs described interactions that addressed logistical and surface aspects of instruction, eight also described at least one interaction focusing on the underlying functions of the new curriculum. These eight teachers reported on participating in discussions that centered on the pedagogical and/or mathematical reasoning behind the new curriculum. Clarissa, a Grade 1 teacher, who said she had “math anxiety” her entire life, remarked how initially *Investigations* was “very confusing . . . coming from the old school way of [teaching mathematics] ‘what do you mean there’s no workbook?’” Acknowledging that her beliefs “have been changing slowly,” she recalled that “the first couple years” of teaching the curriculum as “fake it ‘til you make it. ‘I can’t believe I’m teaching it this way.’” Yet she described how her PLC was

able to work through and do a lot of questioning and wondering about “Why are we doing it this way?” It took probably two years to feel confident that we were being successful because it didn’t make sense to us because it’s not the way we had learned . . . now we see the benefit.

Clarissa described going beyond a focus on teaching strategies in her interactions with colleagues to engaging with the underlying rationale for a more reform-oriented approach. She also discussed seeking out her PLC peers to help her make sense of the new curriculum: “If I had a question about what was coming up I’d go to Dan because he could tell me the why behind it or John ‘why are we doing this? Because this makes no sense to me.’” Similarly, Jessica, a Grade 2 teacher, reported talking about colleagues who “think differently about math,” sharing that:

We’ll even have discussions about how I would add numbers and how she would add numbers, and I go “Well I would do it this way”; exactly how *Investigations* shows you. She goes “not me.” So now I understand why I should teach it another way too . . . it makes me consider different ways of thinking about them [problems] and reasoning about them and not necessarily thinking it was wrong when somebody’s taking a different way.

These eight teachers described at least one interaction with peers that went beyond surface features of instruction to address the underlying pedagogical and/or mathematical reasoning behind reform-oriented instructional activities or strategies.

In summary, and consistent with our quantitative analysis, teachers who interacted with colleagues with more reform-oriented beliefs tended to develop more reform-oriented beliefs themselves, whereas teachers who interacted with colleagues with less reform-oriented beliefs tended to develop less reform-oriented beliefs. Further, and extending our quantitative findings, we theorize based on our qualitative data analysis that the depth and substance of teacher interactions were also important to whether and how they changed their beliefs about mathematics instruction. Specifically, teachers who developed more reform-oriented beliefs reported participating in interactions with peers that focused on deeper pedagogical and/or mathematical principles compared with those who did not.

Discussion and Conclusion

Our account makes several contributions. First, documenting that a majority of teachers across all elementary schools in one school district developed more reform-oriented instructional beliefs over time, we show that change in teachers’ instructional beliefs is possible at scale. Prior work suggests that teachers’ instructional beliefs are difficult to change, and research that does document change in teachers’ mathematical instructional beliefs typically involved small numbers of teachers in intensive and voluntary professional development. Second, we show that teachers’ access to peers’ beliefs in one year was significantly associated with their beliefs the next year, providing additional evidence on the importance of teacher peer

effects (Jackson & Bruegmann, 2009; Penuel et al., 2012). Our account thus helps to open up the black box of teacher peer effects, showing that one way in which peers matter is in the extent to which they influence each other's instructional beliefs. Third, we show that teachers were *not* more likely to interact with like-minded peers who held similar beliefs about mathematics instruction.

This latter finding differs from some prior research (Coburn, 2001) that revealed a tendency for teachers to interact about reading instruction with colleagues who shared their beliefs about reading instruction. There are two possible explanations for this. First, our work focuses on mathematics, and there is evidence to suggest that how elementary teachers think about instruction and its improvement and how they interact with colleagues differs depending on the school subject (Spillane & Hopkins, 2013; Stodolsky, 1988). For example, some research suggests that elementary school teachers are more likely to seek out advice from peers about English language arts than mathematics (Spillane & Hopkins, 2013). Second, prompted in part by the standards-based reform movement, some school districts have worked over the past several decades to redesign their educational infrastructures in ways that foster instructional improvement via teacher professional learning. Thus, educational infrastructures are often intentionally designed to influence with whom and how teachers interact about instruction, and several studies suggest that these efforts do influence teachers' instructional interactions (Atteberry & Bryk, 2010; Coburn, Mata, & Choi, 2013; Coburn & Russell, 2008; Spillane, 2006; Spillane et al., 2015; Spillane, Kim, & Frank, 2012; Stein & Coburn, 2008; Supovitz, 2006). Hence, system and school educational infrastructure design may address teachers' historical dearth of collaboration and support on instructional matters, and therefore teachers' deliberate selection of peers for instructional advice figures less prominently in their interactions about teaching and learning.

A fourth contribution, prompted by conflicting evidence from prior research on the potency of teacher interactions in enabling change, is that we theorize about *how* peer interactions were (and were not) associated with changes in teachers' beliefs. We argue that Auburn Park's redesigned educational infrastructure created opportunities for district and school staff to participate in boundary practices that enabled change in their mathematics instructional beliefs. District leaders working together with coaches and teacher leaders in a district leadership committee developed common understandings of the new reform-oriented curriculum and created boundary objects (e.g., curricular materials, assessments) that were used by teachers as they negotiated their understanding of the new curriculum and its entailments for their teaching. In grade-level PLCs, math coaches, teacher leaders, and teachers participated in discussions with and about the new curriculum and its aligned student assessments, designing lessons, and discussing

students' mathematical work. These boundary practices brought different communities of practice together to make sense of the new curriculum, and this work was facilitated by boundary spanners who connected schools with the district office.

Overall, our account shows how core components of the district's educational infrastructure for elementary mathematics education interacted to support teachers' efforts to implement a more reform-oriented approach to teaching mathematics. While the *Investigations* curriculum was a key component of Auburn Park's infrastructure redesign efforts, disrupting existing practice for teachers and serving as a boundary object that bridged different communities of practice, it was just one component. This is important because too often education research seeks simple and singular panaceas—new curricula, coaching, high-stakes accountability, and so on—failing to examine the multifaceted nature of the educational infrastructure. Our analysis suggests that while the implementation of the *Investigations* curriculum was consequential for teachers, it worked in interaction (rather than in isolation) with other components of the educational infrastructure, including leadership positions and organizational routines that supported boundary spanning and boundary practices.

Still, roughly one-third of teachers in Auburn Park developed less reform-oriented beliefs about mathematics instruction, somewhat surprising given how the district's educational infrastructure was redesigned to support and sustain engagement in boundary practices that supported more reform-oriented beliefs. Based on an analysis of our qualitative data, we theorize that these teachers were less likely to interact with peers who held more reform-oriented beliefs and to engage in substantive interactions with peers about mathematics instruction that addressed underlying mathematical and/or pedagogical rationales for adopting a reform-oriented approach. These conjectures, based on an analysis of data from a small and theoretical sample of teachers, should be explored more thoroughly in future research.

Limitations

With respect to “internal generalizability” (Maxwell, 2005), while our study has numerous strengths (e.g., census of schools and staff, high response rates, purposeful sample of schools and school staff for interview component), there are also limitations. First, there are inherent limitations when using statistical models for inference; it is possible that unobserved factors influenced teachers' patterns of mathematics instructional interactions that we did not measure in this analysis. Second, while our focus was on measuring teachers' beliefs about mathematics instruction, other forms of beliefs might reveal issues of remaining challenge in implementing instructional reform in mathematics, for example beliefs related to students' capabilities to take up principled approaches to mathematics (Jackson, Gibbons, & Sharpe, 2017).

Third, we relied on teachers' self-reports to gauge the quality of their interactions as we did not directly observe them interacting with one another.

With respect to external generalizability (Maxwell, 2005), because our findings are based on a single school district case study, we make no claims for statistical generalizability of our findings (Yin, 2005). Still, our findings have relevance beyond our study site, and we generalize particular findings in several ways: First, taken together, our findings offer a sort of "existence proof" that school districts can (re)design their educational infrastructures to support teachers' interactions with peers about instruction in ways that, on average, are associated with the development of more reform-oriented mathematics instructional beliefs. Our findings, together with those from a few other studies (e.g., Stein & Coburn, 2008), document what is possible (Eisenhart, 2009). At the same time, we acknowledge that the conditions in our case study district are not representative of U.S. school districts in general and therefore turn to the notion of "transferability" (Lincoln & Guba, 1985), arguing that consumers of our findings need to carefully consider the similarities and differences between the district presented here and the school district(s) to which they want to apply our findings. Our findings may not generalize to larger urban school districts and/or districts where teacher turnover is high as such turnover might threaten infrastructure development and the institutionalization of boundary practices. Further, we remind readers that leadership for mathematics instruction in Auburn Park was distributed across central office staff and teacher representatives from most elementary schools; district leaders developed a shared vision for mathematics instruction, worked to design and implement an elaborate educational infrastructure for mathematics instruction to support the realization of that vision in classrooms, and had an appreciation for providing school leaders and teachers with opportunities to learn about mathematics instruction. Research suggests that these circumstances are not the norm in U.S. school districts; thus, we urge readers to exercise care when drawing inferences from our findings for their own contexts.

Second, we can think about the findings reported here in terms of "theoretical" or "analytical" generalizability, which centers on making claims about processes (rather than characteristics of a site or study participants) that may be generalizable outside of a particular study site and contribute to refining existing theories on peer interactions in school systems (Becker, 1990; Eisenhart, 2009; Small, 2009; Yin, 2005). Our work not only replicates but also refines theory on the influence of peer interactions in several ways by showing that (a) one way in which peer interactions may influence performance is by influencing teachers' beliefs about instruction, (b) school district leaders' design decisions with respect to educational infrastructure influence teachers' interactions about instruction by creating and supporting boundary practices that are enabled by boundary spanners and boundary objects, and (c) the depth of interactions among peers may be

especially important when it comes to influencing teachers' instructional beliefs about instruction. So while acknowledging that creating the conditions we described in Auburn Park Public Schools may be difficult, we also claim that should the conditions of interaction we document exist, our findings are likely to hold because we are focusing on a process of broad applicability—peer interactions in the workplace (Becker, 1990).

Practical Implications for School Districts

Our account offers some suggestions for school districts working to transform instruction. First, settling on a vision for instruction is foundational to any effort to redesign an educational infrastructure for supporting instruction and its improvement. Without an agreed on instructional vision, it is difficult to build an infrastructure to support instruction and its improvement in the first place. Second, though standards or curriculum and student assessments have dominated conversations about improving instruction, especially at the state and federal levels, our analysis shows that building an educational infrastructure to support instruction and instructional improvement involves much more, including formal positions and organizational routines. Third, thinking systemically about how various components of an educational infrastructure work together (or not) to influence teachers' interactions about instruction and its improvement and the focus and depth of these interaction is essential. Focusing on one or two components (e.g., curriculum, coaching) in isolation fails to take advantage of how components of the educational infrastructure work in interaction to structure teachers' peer interactions about instruction. Overall, our account offers some cause for optimism related to (re)designing school districts' educational infrastructures to shape teachers' interactions and support reform-oriented instructional beliefs at scale and provides some guidance for such efforts.

Appendix

Beliefs Items, Factor Loadings, and Reliability of the Scales

Please Indicate the Extent to Which You Agree With the Following Statements	Factor Loadings
Factor 1: How Students Should Learn Mathematics ($\alpha = .81$)	
Recall of number facts should precede the development of an understanding of the related operation.	.76
Students should master computational procedures before they are expected to understand how those procedures work.	.82
Time should be spent practicing computational procedures before students are expected to understand the procedures.	.75
Students should not solve simple word problems until they have mastered some number facts.	.69
Time should be spent practicing computational procedures before students spend much time solving problems.	.71
Students will not understand an operation until they have mastered some of the relevant number facts.	.74
Factor 2: How Teachers Should Facilitate Mathematics Instruction ($\alpha = .76$)	
Teachers should encourage students to find their own solutions to math problems even if they are inefficient.	.77
Teachers should allow students to figure out their own ways to solve simple word problems.	.73
The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	.74
Most students can figure out ways to solve many mathematics problems without any adult help.	.63
Mathematics should be presented to children in such a way that they can discover relationships for themselves.	.66
Other	
Most students have to be shown how to solve simple word problems.	
Students need explicit instruction on how to solve word problems.	
Students learn math best by attending to the teacher's explanations.	
To be successful in mathematics, a student must be a good listener.	
Teachers should model specific procedures for solving word problems.	
Students should understand computational procedures before they master them.	
Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution.	

Notes

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¹We appreciate the suggestion from an anonymous reviewer to use a "communities of practice" framework in analyzing our qualitative data and his or her extensive feedback on our usage of the framework.

²The state was one of the last to adopt statewide standards and is one of the few that did not adopt Common Core Standards.

³Results using the case-wise deleted sample were compared to results using multiple imputation for missing data. Because results were the same, we report only the former.

⁴Based on prior survey iterations, we found that limiting the list to 12 did not lead to significant omission of network actors. In the 2013 survey administration, for example, respondents listed an average of 6 people from whom they sought advice or information related to mathematics, ranging from 3 to 10.

⁵Given that network ties can be considered states as opposed to discrete events that have a tendency to persist over time (Snijders, van de Bunt, & Steglich, 2010), our assumption was that teacher *i*'s access to teacher *j*'s beliefs occurred between Time 1 and Time 2.

⁶We assumed that the residuals were independently drawn from a bivariate normal distribution with mean 0, variances σ_0^2 and σ_1^2 , and covariance σ_{01} . They are also uncorrelated with the Level 2 predictor, years of experience, and are homoscedastic over all values of experience. We also assumed that ϵ_{ij} was independently drawn from a normal distribution with mean 0 and variance σ_e^2 and that it was uncorrelated with the Level 1 predictor YEAR and are homoscedastic across occasions.

⁷We used a measure of dissimilarity, or difference in beliefs, in our models; however, for ease of interpretation, we used the term *similarity* to report findings.

⁸Despite conducting cross-sectional analyses, we chose hierarchical latent space models (HLSMs) because they can accommodate covariate effects and allowed us to model several school networks concurrently. HLSMs assume that networks are independent of one another and treat the networks as isolated. Within each network, ties are modeled as independent conditional on the covariates included in the model and the latent positions that facilitate the interpretation of covariate effects.

⁹Goodness-of-fit statistics (i.e., Akaike and Bayesian Information Criterion or AIC and BIC) suggested that Model C is a slightly better fit than Model B.

¹⁰We also examined results for teachers' close colleague interactions. The results were the same.

¹¹On the logit scale, the difference between 3.67 and 4.56 is quite small as both coefficients indicate a very large increase in the probability of a tie.

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